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 December 28, 2024  
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In the present note we will discuss some mistakes in [1].

We begin observing that an  $h^{-2}$  factor is missing in equation (32), since in this equation we state that we can control

$$\sum_{K \in \mathcal{T}_h} \hat{C} \varepsilon \|f\|_{L^2(K)},$$

using  $\|f\|_{L^2(\Omega)}$ . Clearly, this can not be done because the norm of  $f$  inside the sum is not squared. We meant to control each term of the sum by  $\|f\|_{L^2(\Omega)}$  and observe that we are summing over approximately  $O(h^{-2})$  terms, therefore equation (32) reduces to

$$\|u - \hat{u}_h\|_h \leq \inf_{\hat{v}_h \in \hat{V}_{h,\varepsilon}} \|u - \hat{v}_h\|_h + \sum_{K \in \mathcal{T}_h} \hat{C} \varepsilon \|f\|_{L^2(K)} \leq \inf_{\hat{v}_h \in \hat{V}_{h,\varepsilon}} \|u - \hat{v}_h\|_h \quad (1)$$

$$+ \hat{C} h^{-2} \varepsilon \|f\|_{L^2(\Omega)}. \quad (2)$$

Since the term  $\hat{C} h^{-2} \varepsilon \|f\|_{L^2(\Omega)}$  appears already in (36) as a consequence of (35), the overall result is not affected by this correction.

In equation (35) we miss-typed the standard VEM interpolation error estimate, using max instead of min. The correct formula is

$$\|u - \hat{\Pi}_k u\|_h \leq \|u - \Pi_k u\|_h + \|\Pi_k u - \hat{\Pi}_k u\|_h \quad (3)$$

$$\leq C(\Omega) h^{\min\{k,m-1\}} |u|_{H^m(\Omega)} + \|f\|_{L^2(\Omega)} \hat{C} h^{-2} \varepsilon. \quad (4)$$

This correction leads to correcting also equation (36) as follows

$$\|u - \hat{u}_h\|_h \leq C(\Omega) h^{\min\{k,m-1\}} |u|_{H^m(\Omega)} + \|f\|_{L^2(\Omega)} \hat{C} h^{-2} \varepsilon. \quad (5)$$

## References

- [1] Trezzi, M. and Zerbinati, U., 2024. When rational functions meet virtual elements: the lightning virtual element method. *Calcolo*, 61(3), p.35.