Rational functions meet virtual elements: The lightning VEM

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https://arxiv.org/abs/2308.03560

Numerical Analysis in the 21st Century,
15st August 2023, Oxford

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Mathematics

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$$\varepsilon \Delta u \qquad\qquad = f$$

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- **reaction**, the **source** or **sink** of **chemical species** depending up on the **concentration** of the **chemical species**, by the constant γ.

$$\varepsilon\Delta u + \vec{\beta} \cdot \nabla u + \gamma u = f$$

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Advection-diffusion-reaction problem

Multiplying by a test function and integrating by parts we find the **weak formulation** of the **advection-diffusion-reaction** problem, i.e. find $u \in H_0^1(\Omega)$ such that for all $v \in H_0^1(\Omega)$,

$$\varepsilon \int_\Omega \nabla u \cdot \nabla v \, d\vec{\boldsymbol{x}} + \int_\Omega (\vec{\boldsymbol{\beta}} \cdot \nabla u) v \, d\vec{\boldsymbol{x}} + \gamma \int_\Omega uv \, d\vec{\boldsymbol{x}} = \int_\Omega fv \, d\vec{\boldsymbol{x}}.$$

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We can rewrite this problem in compact form, using the bilinear form $a(\cdot,\cdot) : H_0^1(\Omega) \times H_0^1(\Omega) \to \mathbb{R}$, i.e. find $u \in H_0^1(\Omega)$ such that for all $v \in H_0^1(\Omega)$,

$$a(u,v) := \varepsilon \left(\nabla u, \nabla v\right)_{0,\Omega} + \left(\vec{\beta} \cdot \nabla u, v\right)_{0,\Omega} + \gamma \left(u, v\right)_{0,\Omega} = \left(f, v\right)_{0,\Omega}.$$

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- The **advection-diffusion-reaction problem** can't be solved using the standard **lightning Laplace** method.
- We can easily return to the **Laplace problem** and **diffusion-reaction problem** changing the parameter γ and $\vec{\beta}$.
- The **lightning VEM** method will allow for a simpler construction than the **vanilla VEM**.

Conforming Galerkin method

We first consider a **conforming discrete space**, i.e.

$$V_h = \langle \phi_1, \ldots, \phi_N \rangle \subset H_0^1(\Omega), \qquad \dim(V_h) = N$$

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We are left solving a **linear system** to find the value of the coefficients $\vec{\boldsymbol{U}}_i$, representing u_h in the chosen base, i.e.

$$A\vec{\boldsymbol{U}} = \vec{\boldsymbol{F}}, \qquad u_h = \sum_{i=1}^{N} \vec{\boldsymbol{U}}_i \phi_i.$$

Finite element method

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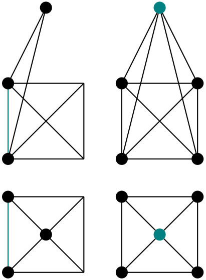
To solve the previously mentioned problem we turn to a **finite element** $(K, V_h(K), \Sigma)$ discretisation, i.e.



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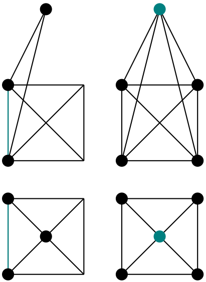

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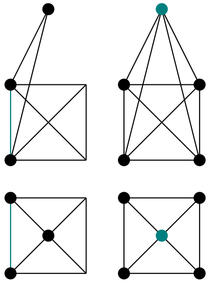

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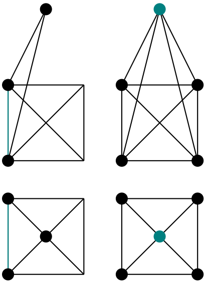

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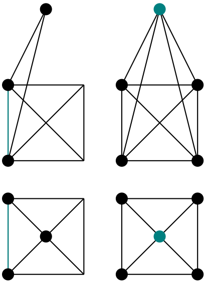

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- We need to determine the **connectivity** of the **DOF**.

Failure point of the FEM: mesh types

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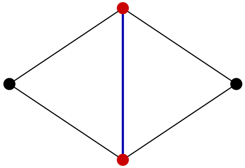

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The red **DOF** ensures the **continuity** across the blue edge, hence H^1 **conformity**.

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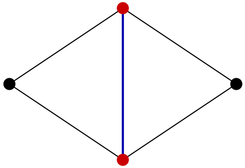

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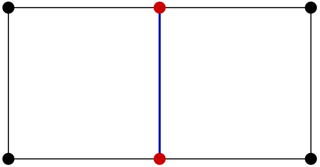

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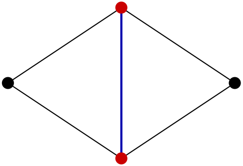

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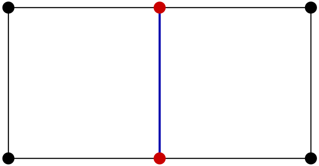

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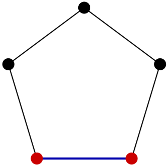

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How can we deal with a **general polygon** ?

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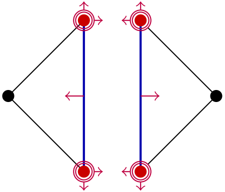

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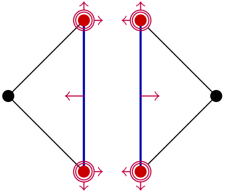

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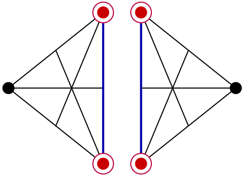

On **Powell–Sabin splits**, we can decrease the polynomial order to needed for $\mathcal{C}^1$ **conformity**.

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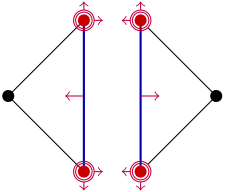

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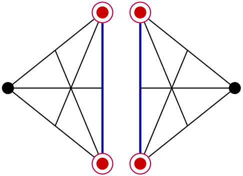

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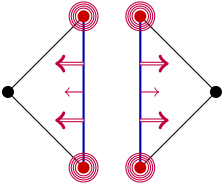

The **Bramble–Zalmal element** is $\mathcal{C}^r$ **conforming**, and requires degree $4r+1$.

Free lunch: The virtual element method

The virtual element method is based on fixing the degrees of freedom we need on each element's edge and constructing basis functions that can be determined starting from these degrees of freedom.

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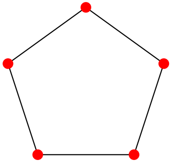
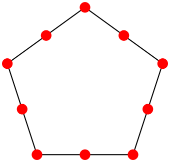


$k = 1$



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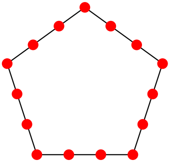

$k = 3$

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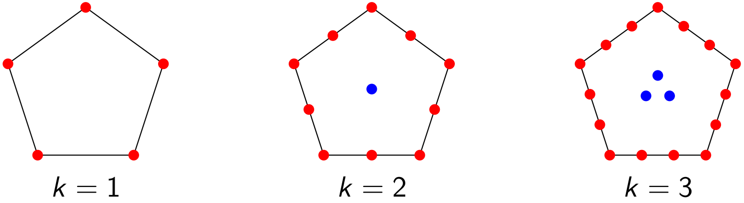

$$V_h(K) := \left\{ v_h \in H^1(K) \ : \ \Delta v_h \in \mathbb{P}_{k-2}(K) \ \text{and} \ v_h\big|_e \in \mathbb{P}_k(e) \right\}$$

The cashier is shouting at us !

The **discrete variational problem**, find $u_h \in V_h$ such that for all $j = 1, \dots, N$

$$a(u_h, \phi_j) = \sum_{i=1}^{N} \vec{\boldsymbol{U}}_i a(\phi_i, \phi_j) = (f, \phi_j)_{0,\Omega},$$

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requires us to solve a **Laplace problem** on each element:

$$\begin{aligned} \Delta\phi_i &= \omega_i \ \text{ in } K, \\ \phi_i &= \varphi_i \ \text{ on } \partial K. \end{aligned}$$

where ω_i are the basis function corresponding to the **internal DOF** and φ_i are the basis function corresponding to the **edge DOF**.

We run away: The projector operator

We can construct the entries of the matrix A using **only the DOF**!

$$\Pi_k^{\nabla,K} : V_h(K) \to \mathbb{P}_k(K),$$

$$\int_K \nabla p_k \cdot \nabla(\phi - \Pi_k^{\nabla,K}\phi)\,\mathrm{d}K = 0, \qquad \int_{\partial K} (\phi - \Pi_k^{\nabla,K}\phi)\,\mathrm{d}s = 0\,.$$

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Now we **break** the bilinear form on each element of the tessellation, and starting from the **diffusion** term observe:

$$\varepsilon \sum_{K\in\mathcal{T}_h} (\nabla\phi_i, \nabla\phi_j)_{0,K} = \varepsilon \sum_{K\in\mathcal{T}_h} (\nabla\Pi_k^{\nabla,K}\phi_i, \nabla\Pi_k^{\nabla,K}\phi_j)_{0,K}$$
$$+\,\varepsilon \sum_{K\in\mathcal{T}_h} (\nabla(I - \Pi_k^{\nabla,K})\phi_i, \nabla(I - \Pi_k^{\nabla,K})\phi_j)_{0,K}$$

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$$+ \varepsilon \sum_{K\in\mathcal{T}_h} S\Big((I - \Pi_k^{\nabla,K})\phi_i, (I - \Pi_k^{\nabla,K})\phi_j\Big)_{0,K}$$

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- Adding a projector operator for the **advection term** naively will result in a **non-skew-symmetric** system !
- We only have access to the value of the **DOF**. How do we access the **point-wise** value of the solution ?

TESCO Budget meal: The lightning VEM

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Our idea is to solve **cheaply** and **accurately** solve the **Laplace problem**,

$$\begin{aligned} \Delta\phi_i &= \omega_i \ \text{ in } K, \\ \phi_i &= \varphi_i \ \text{ on } \partial K. \end{aligned}$$

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- **curved mesh elements**, resorting to the **AAA** method.

The lightning Laplace method

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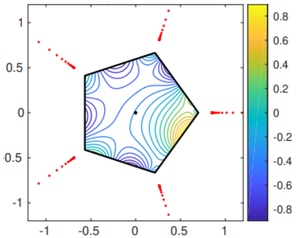
The idea behind the **lightning Laplace** method is to construct a solution to the **Laplace equation** of the form,

$$\hat{\phi}_i = \mathrm{Re}\left\{ \sum_{j=0}^{N_P} \frac{a_j}{z - z_j} + \sum_{j=0}^{N_Z} b_j (z - z_*)^j \right\},$$

where $\{z_j\}_{j=1}^{N_P}$ and z_* are points in the complex plane and Re denotes the real part of a complex number.



Non-Conforming Galerkin methods

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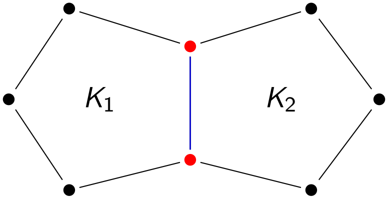
We know that the basis function $\hat{\phi}_{i,K_1}$ and $\hat{\phi}_{i,K_2}$ corresponding to the i-th vertex and constructed respectively on K_1 and K_2, match at the degrees of freedom here denoted in red.



Non-Conforming Galerkin methods

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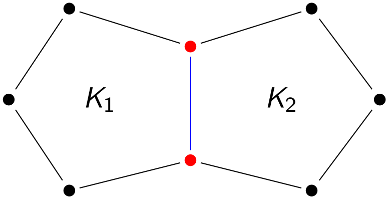

Yet we have no guarantee that $\hat{\phi}_1$ and $\hat{\phi}_2$ are continuous along the blue edge.

Non-Conforming Galerkin methods

We begin introducing a larger space, i.e. $V = H^1_0(\Omega) + V_h$ and observing that the **broken** bilinear form has meaning on V, i.e.

$$a_h : V \times V \to \mathbb{R}$$

$$a_h(u, v) = \sum_{K \in \mathcal{T}_h} \varepsilon(\nabla u, \nabla v)_{0,K} + \left((\vec{\beta} \cdot \nabla u), \nabla v\right)_{0,K} + \gamma(u, v)_{0,K}$$

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- When we consider $a_h(\cdot,\cdot)$ on $H_0^1(\Omega)$ we have that $a_h(\cdot,\cdot) = a(\cdot,\cdot)$
- Thanks to the **DOF** we know $a_h : V \times V \to \mathbb{R}$ is a scalar product, so we can apply **Lax-Milgram lemma** to prove the existence of discrete solution.

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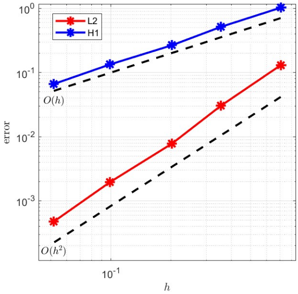
A priori error estimates

Assuming we are solving the **local Laplace** accurately enough we can prove the following a priori error estimates,

$$\|u - \hat{u}_h\|_h \leq C(\Omega) h^{\max\{k,m-1\}} |u|_{H^m(\Omega)} + \|f\|_{L^2(\Omega)} \hat{C}\varepsilon.$$



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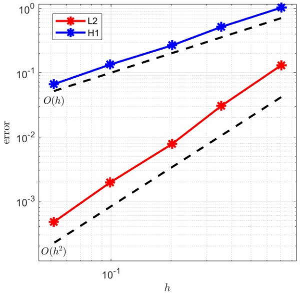
where ε corresponds to the tolerance of our **local lightning Laplace solve** with respect to the $H^{\frac{1}{2}}(\partial K)$ norm.



Conclusion

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Thank you for your attention !

Yes but: Performance of the Lightning VEM

Table: A comparison between a vanilla VEM implementation and the lightning VEM implementation, of the average time (in seconds) taken by the assemby of the local matrix for different numbers of elements.

N	4	16	64	256	1024
Vanilla	4.61e-03	2.03e-03	2.20e-03	1.10e-03	1.03e-03
Lightning	3.67e-03	3.22e-03	6.07e-03	9.15e-03	1.84e-02